How do students experience open-ended math problems?

An Action Research Project

by

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Abstract

The study of math is often viewed as the memorization and practice of a closed set of rules and procedures. This outlook stands in stark contrast with the mathematical needs of a person entering the job market in virtually any field. My study examines what happens in a math classroom when students work on open-ended problems, i.e. problems with multiple methods or multiple solutions. By analyzing work samples and surveys of 56 students and interviews of six focus students at High Tech Middle, a project-based school in Southern California, students’ experiences shed light on the effectiveness of these problems as tools in a math classroom. It was found that open-ended problems helped students to have a more positive view of themselves as mathematicians. Open-ended problems also helped to increase students’ problem-solving skills and were helpful in challenging students at all levels of mathematical understanding. These findings indicate that these problems may be valuable in helping increase student confidence and achievement in math classrooms everywhere.
Introduction

In my first student teaching placement, I entered a very traditional math classroom. Emulating my master teacher, I would cover a section out of the text book each day, give homework for that night, review the next day, and then move onto the next section. Tests were multiple choice scantron forms testing the students’ ability to memorize the rigid procedures covered in a given chapter before moving on to the next set of topics. Rarely were any of the topics taught with any overlap to each other and even less frequently were the students asked to apply any of the formulas and algorithms they had memorized to a meaningful situation. While my experience with students there was positive, I left feeling like I could and should have done more. I also felt very strongly that the way I was teaching math was stripping it of its beauty and relevance. When I began working at High Tech Middle, I had quite the opposite experience. Not only did I not have a textbook to teach out of, I was free to teach math as I saw fit. I saw this as both an exciting and daunting task; one carrying lots of responsibility. I did not take this responsibility lightly and, since my hiring two years ago, I have been able to try many different types of math activities, lessons, games, and even online tutorials. I have experimented with group problems and quizzes, student presentations, oral assessments, traditional lecturing, and kinesthetic math activities. I have found some that work well, and others that seem to fail miserably. Through all of these activities, I struggled to challenge the advanced math students, accelerate the ones who lacked basic skills, and preserve the beauty of discovering math that intrigued me when I was in middle school.

Throughout the end of my first year of teaching and into summer school, I became acquainted with a well-respected math education consultant who had decided to work with High Tech Middle and their math program. Through work with him, I became tuned in to open-ended
math problems. I was intrigued by the idea of giving my students problems to solve whose answers weren’t immediately evident even to me, a person with a degree in math! These problems were accessible to students of all ages, but also challenged students of all ages. I was also intrigued by the amount of content that could be covered, discovered rather, through one exploratory problem. As I worked on these problems with my summer school students, I began to see the beauty and intrigue of math again and began to feel inspired about new ways to meet the needs of all of my students. I felt that these types of problems allowed me to teach math with relevance, through multiple strategies, and through discovery. They allowed me to challenge the ones ready for it, and help develop the confidence of the ones in need of support.

After working with these problems for a few months, I felt that I was continually learning more about them; however, I wanted to challenge my own perceptions of them and instead attempt to see the problems through a student lens. I know that a lot of High Tech High teachers have experimented with these types of problems because they seem to go hand in hand with our design principles of personalization, common intellectual mission, and adult-world connection.

The question I researched was “How do students experience open-ended math problems.” In choosing this, my hopes were that I would unlock some ideas about the usefulness of these problems through students’ eyes and how that corresponds to teachers’ intent with the problem. I also wanted to assess how well these problems were helping all students feel successful in math. Finally, it was my hope to gauge students’ mathematical depth of understanding, in particular how well they apply what they are learning to new situations. Hopefully my findings will open up discussions with other math teachers about the benefits and drawbacks of these problems so that we can attempt to unravel the place they have in the High Tech math classroom.
**The Context**

*Literature Review*

Open-ended math problems, as they pertain to my study, are problems that can be solved in more than one way or have more than one solution (Schuster, 2005). They are generally just beyond the student’s skill level and can be done either collaboratively or individually (Jarrett, 2000). Finally, open-ended problems include situations that require analytical, creative, and critical thinking skills to solve (Forsten, 1992). Some examples of open-ended math problems promoting number sense at the 5-6 grade level are as follows:

“*Why do you suppose a day is split into 24 hours? Do you think another number would have been a better choice? Justify your defense of 24 or your new number proposal with mathematical arguments.*”

“The weather is reported every 18 minutes on NBC and every 12 minutes on Fox News. Both stations broadcast the weather at 1:30pm. *When is the next time the stations will broadcast the weather at the same time? At what intervals of time should they report the weather so that they are never reporting at the same time?*”

(Problems adapted from Schuster, 2005)

While each of these problems involve a situation that would require analytical, creative, and critical thinking skills to solve, the first exemplifies a problem with multiple “solutions” and the second a problem with one answer and many possible approaches.

Advocates of open-ended problems cite three general benefits of open-ended problems: they develop problem-solving skills, they allow for natural differentiation, and they help students make connections across mathematical concepts.
Problem Solving. When the National Council of Teachers of Mathematics (NCTM) released their Curriculum and Evaluation Standards for School Mathematics in 1989 and their follow-up Principles and Standards for School Mathematics in 2000, the aim was to reform math education to better meet the needs of our society. A common focus in each document was the development of problem-solving skills and the application of mathematical concepts to real-world contexts. According to the NCTM, problem solving means “engaging in a task for which the solution is not known in advance. Good problem solvers have a ‘mathematical disposition’—they analyze situations carefully in mathematical terms and naturally come to pose problems based on situations they see” (NCTM, 2000). Open-ended problems support this type of learning because they require students not only to understand the problem, but also to think about how they arrived at their conclusion, thus moving toward the aforementioned "mathematical disposition." They similarly allow students and/or teachers to construct new investigatory questions about the posed situation (Schuster, 2005). Whereas students working on traditional or “closed” math problems might reason with numbers to produce an answer, open-ended problem require students to reason about numbers to produce understanding (Steen, 2007). According to Robert McIntosh, a Mathematics Associate for the Northwest Regional Educational Laboratory, "To develop these (problem solving) abilities, students need ample opportunities to experience the frustration and exhilaration that comes from struggling with, and overcoming, a daunting intellectual obstacle" (quoted by Jarrett, 2000, p. 5). Open-ended problems provide this opportunity as students work on their own to unravel their misconceptions and determine the best procedure for solving the problem (Schuster, 2005).

Successful problem solvers have a plethora of tools that they use to be successful. Well-respected mathematician, George Pólya outlines a process for problem solving that follows the
In her book about teaching problem solving in math, Char Forsten states that problem solving strategies are “plans of action” that include making tables and organized lists, working backwards, findings patterns, and guess and check (Forsten, 1992). Most problems proposed throughout the book to teach such pattern recognition are open-ended either in process or solution.

**Differentiation.** Open-ended problems are advocated as excellent tools for both gifted/talented students and struggling students and are therefore an excellent tool for differentiation, particularly in a detracked classroom.

As Hertzog observes, open-ended problems allow students to work in their own learning styles and at their own ability levels, making personal choices in their process. They also include options for students to interact with content and to elaborate and integrate knowledge across disciplines. Hertzog writes, “Curricula for the gifted/talented should focus on and be organized to include more elaborate, complex, and in-depth study of major ideas, problems, and themes” (Hertzog, 1998). Open-ended math problems focus on major themes and offer many opportunities for student or teacher driven extensions and therefore cater perfectly to the needs of gifted students.

When it comes to struggling students, open-ended problems offer increased emphasis on problem-solving and reasoning with a decreased emphasis on the correct answer. Open-ended problems based on real-life situations have been found to help struggling students have a better opinion of math and a higher self-esteem (White, 1997). A San Francisco Bay Area study of a group of disadvantaged urban high school students found that learning through these types of
problems not only produced more positive feelings about math, but the students in these mixed-ability, de-tracked classrooms outperformed wealthier teenagers in tracked, traditional classrooms on various assessments (Trei, 2005).

Appealing both to the gifted and the struggling, the multiple entry points of open-ended problems can address the needs of each student and help to create a differentiated classroom.

**Connectivity and coherence.** One of the beauties of math is the interconnectedness of topics. While fractions, decimals, and percents can represent the same value, each will be useful in a different context. A traditional math textbook may contain upwards of 15 chapters with six or more sections in each chapter, each section representing a different topic or subtopic. Teaching straight through a textbook may exchange the beauty and connectedness of mathematics for rote memorization of seemingly disconnected topics. Open-ended problems encourage students to make connections because these problems often cover many different topics all within one problem. Because of this, not only are the students applying facts and procedures that they may have learned in the past, but they are making connections and generalizations with what is going on in the problem (Schuster, 2005). Often, students are reflecting on their own work and collaborating with others as they work which helps them to make even more connections as they see other students’ approaches to the problem. Through this reflection and collaboration, the student constructs an understanding of the problem that is both deep and flexible because prior knowledge is connected with new concepts and skills (Jarrett, 2000).
Why examine students’ experiences?

Alan Schoenfield, in his publication called “The Math Wars” outlines the historical plight of mathematics curriculum in the United States. Following an almost sinusoidal motion, the focus of math before and after World War 2 has swayed between a focus on problem solving and a “back to basics” approach. Because of these differing opinions, a lot of attention was drawn to math curriculum and by the 1980’s, a fair amount of research had been done on “mathematical competence” and what this looks like. In addition to having strong problem solving skills, the mathematically competent, “have a strong knowledge base, make effective use of the knowledge one has (metacognition) and has a set of productive beliefs about oneself (self-efficacy)” (Schoenfield, 2000).

Though rarely touted as emotive, math is a subject that evokes plenty of feelings both positive and negative in students. By the time they have arrived in middle school, many students have already dubbed themselves (or have been dubbed) “good” or “bad” math students. These feelings that they bring into the classroom either set them up for success, or build a wall leading to ultimate failure. Students’ experience in the classroom is often tied to their self-efficacy that Schoenfield discussed in “The Math Wars”. A students’ self-efficacy is their belief that they are capable in a specific subject matter, in this case, math. Pajares and Kranzler found that a students’ self-efficacy in math is as predictive as a students’ general mental ability (Pajares & Kranzler, 1995).

Further research has shown a correlation between self-efficacy and successful problem solving in the math classroom. Bandura found that self-efficacy is a highly predictive indicator of motivation and learning (Bandura, 1997). Furthermore, self-efficacy is highly predictive of
mathematical problem solving and is significantly related to students' selection of science-based college majors (Betz, 1983; Pajares, 1994).

Because self-efficacy is such a strong indicator of success in a math classroom, it is of utmost importance that I am keeping an eye toward my students' feelings about themselves and about mathematics throughout this study. It is only in understanding students’ feelings as they pertain to my classroom experiences that I can begin to unravel ways that our learning environment can be improved. Though some educators have become aware of the benefits of open-ended problems, studies surrounding students’ subjective views about their effectiveness have been limited. The purpose of this study, therefore, is to make my students’ feelings and experiences the center of my data collection and to seek to determine the success of these problems when investigated through the lens of a student.

Description of the Setting

School Setting

I implemented open-ended math problems in two untracked 6th grade math/earth science classrooms at a public charter school in San Diego, California called High Tech Middle. High Tech Middle is part of an organization that includes five high schools and two middle schools located in Point Loma, Chula Vista, and North County. All of these schools are project-based and technology driven. High Tech Middle serves roughly 350 students in grades six through eight. Enrollment is non-selective and High Tech Middle does not consider grades or recommendations when admitting students. Admittance is instead based on a lottery system in which students are selected by zip code in an attempt to ensure a diverse student body. Students
with a sibling at one of the schools are given priority over other applicants as do students entering from the elementary school that is also affiliated with these schools, Explorer Elementary. There is an extensive waiting list to get into the school. Through these processes, High Tech Middle attempts to attain a student body that is representative of San Diego Unified School District. High Tech Middle gets closer to district demographics each year but does not yet mimic district ethnicity percentages as seen in the table 1 below.

<table>
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<tr>
<th></th>
<th>Enrolled</th>
<th>% Asian</th>
<th>% Black</th>
<th>% White</th>
<th>% Filipino</th>
<th>% Hispanic</th>
<th>% FRL</th>
<th>% Special Ed</th>
<th>% ELL</th>
</tr>
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<td>6</td>
<td>12</td>
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<td>30</td>
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<td>1</td>
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<td>13</td>
<td>25</td>
<td>7</td>
<td>44</td>
<td>57</td>
<td>12</td>
<td>30</td>
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As you can see in Table 1, High Tech Middle has significantly less Hispanic students than the district and it therefore has a much smaller percentage of English Language Learners than the district. The percent of students who qualify for free and reduced lunch are 30% in contrast to the district which stands at 57%. 11% of the students at High Tech Middle qualify for Special Education Services which is very close to the district breakdown.

To be entered into the lottery for admittance to the school, parents must fill out an application and attend an informational meeting. As a result, the school attracts students whose parents are more regularly involved in their child’s education. Parents are highly encouraged to participate actively in their child’s education, coming into school multiple times throughout the year for parent teacher conferences, exhibitions of student work, formal presentations of learning (at each semester’s end), field trip driving, fundraisers, monthly forums held by the director, and even family math night!
The average class size at High Tech Middle is 28 students and the special needs program is full inclusion (there are no separate special education classes). This is in contrast with the San Diego Unified School District which often has separate special education classes and pull-outs for students with an Individualized Education Plan (IEP). Teachers, with support of grade-level tutors, support the diverse range of special needs and IEP students within the classroom through highly differentiated curricula and projects.

High Tech Middle has a very specific educational philosophy embodied in three core values: personalization, adult-world connection, and common intellectual mission.

Personalization at High Tech Middle is provided to students on many levels throughout their time at the school. Core teachers teach only two block classes a day (a maximum of 56 students a year). Teachers are on two-person interdisciplinary teams and these two teachers share the same students throughout the entire year. The positive implication of this is that teachers have fewer students whom they then have the opportunity to know well. Students also have one faculty member who serves as their advisor for their duration at the middle school.

To support an adult-world connection, teachers regularly host guest speakers and take students on field trips in an attempt to connect their learning to the world around them. Students work on interdisciplinary projects that impact their immediate world such as proposing solutions for the current San Diego water crisis.

High Tech Middle maintains a common intellectual mission which means that technical training and college preparatory work are combined to properly prepare students for the real world. There are about 15 laptops in every classroom at High Tech Middle for students to use both for research and for becoming proficient in various types of software. There is no tracking at High Tech Middle and so students at all different levels and from different backgrounds
interact on a daily basis in each classroom. The technology-driven, project-based approach is one that enhances the learning experiences for this diverse group of students.

Math has often been a subject that has not followed in step with the progress of the school. It has often been taught traditionally using primarily direct instruction and repetitive problem sets. Recently there has been a push amongst teachers to expose students to more content rich, thought-provoking math work. Many teachers attended several training sessions on teaching math through a more open-ended approach and there seems to have been a shift mathematically in the school. Many teachers have also realized that although textbooks offer reinforcement of some core mathematical skills, students’ abilities to think and reason mathematically are often stifled by the closed problems that textbooks offer. These teachers, including me, are working to foster a more positive math experience for the students.

**Classroom Setting**

Implementation of my open-ended problems took place over the course of a semester in my classroom where students frequently learn through investigation, discovery, and discussion. Open-ended problems were a part of our daily activities including warm-up and homework problems. The students in my classroom sit in groups of four and are regularly encouraged to work with one another on their math work. The demographics of my classroom generally match-up with the demographics for the school as a whole. Of my 56 students, five are on IEP’s and each has the support of an in-class tutor.

In a pre-assessment administered during the first week of school, students in my class performed at varying levels in every subtopic. On average, students in the class scored highest on
their proportional reasoning (80.5% at or above critical level) and lowest on their Data Analysis skills (21.5% of class at or above critical level).

Selected focus students

I selected six focus students for my study, three boys and three girls. Students were selected based on various components including their pre-assessment scores, their journal responses, and ethnicity. It was my goal to have the students represent a cross section of my classroom population in each of these categories. Many of the students I selected were verbose in their journal writing and these students gave me a good picture of what they were experiencing, however I also included two students who initially did not write much in their entries. With these students, I instead sought their experiences out in our interviews.

I selected three boys for my study. The first was Julio, a Hispanic student who lives with both of his parents in a middle class area of San Diego. Julio is a very hard worker and always exercises diligence on all work. He is a very sensitive boy, easily discouraged when he doesn’t understand something in my math class. On his pre-assessment score, he scored 26 out of 40 which is above the average score by a few points. Julio was above the critical level in four of the six mathematical categories and below in Data Analysis and Fractions, decimals, and percents.

The second student I selected was James, an African American student from the outskirts of San Diego. His mom was very involved in the class and regularly volunteered to help with classroom field trips and activities. James scored 14 out of 40 on his pre-assessment score which put him in the lower quartile of students in my class. Interestingly, James regularly comes across as understanding the material, but upon deeper inquiry, it is revealed that he is confused on a
deeper level. James was below the critical level in all six of the math categories on our pre-assessment.

My final boy was a student named Tim, a Caucasian student with many siblings. Tim is a hard worker who is eloquent and honest. Tim does extremely well socially and came from our elementary feeder school, Explorer Elementary. Tim is rather quiet in class however whenever he raises his hand to offer input, it is clear that he has a very deep understanding of everything we are discussing. Tim scored a 33 out of 40 possible points on our pre-assessment in the beginning of the year and was above the critical level of understanding in four of the six categories. He fell below this level in data analysis and integers.

The first girl I selected is Megan. Megan is an African American student who is very socially aware. She is well-liked among her friends in and out of the classroom. In math, Megan struggled with tests. She regularly raises her hand and answers questions, however she scored quite low on our pre-assessment (13/40). I have seen a growing confidence throughout the semester in Megan as she has taken on the new challenges of middle school. Though initially below the critical level in each of the six math categories, Megan is holding her own in class and regularly shares out during discussions and works hard on all projects.

My next spotlight student was Felice, a Filipino student who lives with both parents in Southern San Diego. Felice has been in accelerated Kumon math classes for much of her schooling. She is very bright and hard-working, never submitting work that is less than the highest of quality. On our pre-assessment, Felice scored a 37 out of 40 and was above the critical level in all six categories. She is an eager student and is often willing to help students around her reach their highest potential.
Finally, I have chosen to spotlight Amber, a Caucasian student who struggles with math. Upon speaking with her parents, I found out that Amber has never considered herself to have any math skills and is has become increasingly evident in my class that she doubts herself so much that at times she gives up before she even begins. She is a hard-worker but never quite hard enough to overcome her lack of confidence. On our pre-assessment, she scored 12 out of 40, below the critical level in all five of the six categories. Amber is relatively quiet; however, she does express herself well in writing and so I have been happy to get to know her better through her journal entries.

Through work samples and interviews I hope to better get to know each of these students and gain an understanding of their experience of math in my class where open-ended problems form the base for our curriculum.

**Methods**

*Data Collection*

**Surveys:** In order to unravel the question of “How do students experience open-ended math problems?” I initially did a survey of each of my 56 students to gather quantitative data and seek trends in their general feelings about math and ideas about various types of math problems. The same survey was given in the beginning, middle, and end of my study to seek changes in students’ feelings about these types of problems as the semester progressed. Appendix 1 has my survey in its entirety; however here is a sample question:

1. How would you describe yourself as a math student?
   a) I’m a math genius… give me any problem and I’ll solve it for you.
   b) I’m pretty good at math… it’s my best subject.
   c) Math is alright… I am improving at it.
   d) I don’t think I’m good at math at all
   e) Other ________________
Interviews: My second method of data collection was to conduct interviews. I selected six spotlight students with whom I conducted brief 10-15 minute interviews after school. I selected the students for my focus group based on their math pre-assessment scores in my class and the quality of their reflections on their math journal problems. I also worked to keep the racial make-up of these students similar to the racial make-up of my class. I selected three boys and three girls of varying math abilities and backgrounds and asked them questions pertaining particularly to their experiences on their journal problems. I had the students “prep” for our interview by reviewing the reflections in their journal throughout the semester.

Work Samples: My third method of data collection was student journaling and work samples. The data that I collected through this method provided me with tangible evidence of what kids were able to do and of the range of responses they made to open-ended problems. Having all of their open-ended problems condensed in one journal provided me with a quick way to flip through their work seeking trends and notable exceptions, both of which were quite informative to my research. After each problem, I had the students answer four questions in the form of a journal entry. See Appendix 2 for the journal entry prompts. All 56 of my students participated in this, and I collected and reviewed their journals after each problem not only to grade them, but to look at their various strategies, and extract quotes from their reflections.

Data Analysis

Surveys: The quantitative data that I collected through beginning, mid, and end of the year surveys was entered into excel and I kept track of student identities through a numeric system. I visually interpreted the data using a program called fathom. I used these graphs to track students’
responses to questions with respect to their feelings about themselves as mathematicians and their feelings toward open-ended problems. I sought trends on each survey; however, as I accumulated the mid and final survey data, I tried to cross-examine the responses that either changed or remained the same from one survey to the next for individual students.

**Interviews:** To analyze the data from my interviews, I transcribed the audio files in their entirety. Transcriptions of these sessions helped me to further investigate trends in student thinking and furthermore determine if these problems met the needs of each of my students as my interviewees were carefully selected from all ability levels and backgrounds. I read through these transcriptions looking for various keywords and quotes that pointed toward an understanding of students’ experiences on these problems.

**Work Samples:** About once a week the students worked on a selected open-ended problem in their math journal. I found many of these problems in my resources (see references) and designed some on my own. I worked to gain a deep understanding of the problems so that I might properly extend the problems for students who were prepared for such a challenge and remediate for those in need of extra help. The most basic components of the problems were at a sixth grade level. Duration of work on the problems varied from one-60 minute period to a few days of work with intermittent reflections, extensions, and remediation within the problem. Work on these problems was followed by a guided written reflection where I sought to gather their thoughts on the problem. As I read through their journal responses, I regularly worked to modify and revise future problems to better meet the needs of my diverse student population and challenge each student at his/her own level.
My methods for analyzing student journaling evolved through the course of my data collection. Initially I looked for key words and trends and attempted to color code their journals with post-its. This turned out to be a frustrating task as the post-its routinely fell out leaving me unsure of where the quote was in the journal. I also tried to make copies of particularly telling responses however their writing was not legible with the copy machine. In the end, I decided to make an excel spreadsheet for each journal and type in key quotes as I graded the journals. This helped streamline the process because, when searching for trends, I needed only to scan the spreadsheets for keywords like “patterns” or “enjoy”. This was all to provide me insight to their interpretation of the problem and their ability to take this information to the next level of solving the problem.

**What I Learned**

*Findings and Actions: Part 1 (chronological)*

I will begin my findings by describing first the implementation of each open-ended problem in my classroom. This will include what I saw and heard while the students were working and comments that stood out in their journals. This process of implementation was continually evolving throughout the semester as I listened to the voices of my students, and I similarly will document those modifications in the sections below. For the complete versions of the problems I used, please see appendices 3-13.

In later sections, I will pinpoint the specific themes that I saw throughout this entire process regarding how students experienced these open-ended math problems.

**Problem #1: The Weather Problem** *(see appendix 3)*

For our first journal problem, I used “The Weather Problem,” a problem investigating
relationships between numbers including factors, multiples, and co-prime numbers. It also required the students to work with time which was an unexpected challenge from my standpoint. (Example: what is 45 minutes before 12:15 pm posed a problem for some students). For this problem I allowed the students to work in self-selected groupings. I also gave them a freedom of space meaning I allowed them to choose whether they would work inside the classroom or outside in our commons area. Immediately the space inside the classroom became the “quiet” work environment and the commons area became a space for those who preferred to work more interpersonally. I rotated around both inside and outside the classroom and only a few times did a line of students build up with questions for me. I encouraged them to employ my classroom’s “three before me” policy of asking three other students the question before coming to me.

Overall, the students seemed to really enjoy diving into this problem. I took great care to be positive about getting them started because I know the power of first impressions with an ongoing activity like this and I think the students fed off this care. By the end of our two periods (hours) of work on the problem, about 95% of the students were able to finish the first few parts of the problem and about three finished all of the extensions (perhaps more were needed). This fell short of my expectations that more students would make further progress and get into the idea of “co-prime numbers” but the aforementioned challenges that arose with working with “time” held many students back.

I would like to address the skill of counting in time before beginning the problem next time. Perhaps a warm-up problem asking students something like, “what time is 45 minutes before 12:30?” would better prepare them for success. Another area for improvement was in final presentations of work. Because I was crunched for time, I did not have students put work on the board. Rather, I briefly went over the problem myself though not in depth enough to generate
any real discussion. I think the students who were initially confused were still a little confused in the end. I did not have time to look at all parts and so even students who did get further along were left wondering if they had done the problem correctly. Next time I would like to manage time better and have students put work on the board to explain various parts of the problem to the class. Overall, this was a good start, but I have a lot to work on with my strategies and time management for implementation.

**Problem #2: The Locker Problem** *(see appendix 4)*

The locker problem is a fairly well known problem incorporating many number sense concepts such as factors, multiples, and perfect squares. These concepts are not explicit upon reading of the problem; rather students discover these concepts in their work on the problem. Due to the modified schedule this week, this problem ended up lasting us three days (one hour long period on each day). I started the week with a lesson on diligence. I was able to talk extensively about how real mathematicians frequently work on problems for months and even years! Because the Locker Problem took us so long to complete, the students were able to see their work as similar to the work that of a “real mathematician” and many expressed positive thoughts and feelings about this. By the second and third day of work, the behavior of students who were struggling started to decline because I was not providing ample support to everyone at every moment that they needed it. This would be something that I would work like to work on supporting better next year.

The lessons in mathematical diligence definitely outshined the content knowledge that was gained in this problem. I gave them a locker problem partner quiz at the end of the week and many students did quite well showing that they understood key aspects from the problem.
Nevertheless, one of the key concepts in this problem was factors and perfect squares and about one week after the problem, I had a few students exclaim “what is a perfect square again?” I know this is just terminology and not the main point, but I was thinking that after all of the interaction with the concept on this and other problems, that it would be permanently planted in their brains! On this problem, none of the students completed all of the extensions and some of the students were confused by the wording of the problem, stopping their “diagrams” and work after locker 6 (not noting my “etc.” indicating that the pattern continued through locker 100)

For the students who took the time to think through and understand the problem, this was an awesome experience. Nevertheless, with my current implementation, too many students were left confused leading me to think that this problem needs to be better scaffolded for sixth grade level. This could be done by providing “hint cards” to students who are struggling. Additionally, it is of utmost importance that I make sure that my wording is abundantly clear in each of these problems. Perhaps I will work to create shorter problems with simpler language which would better meet the needs of my ELL students as well. Finally, I would like to explore different strategies for the extension opportunities because on this problem, some students expressed feeling overwhelmed by all of the extensions on the paper that they “would NEVER be able to get to!”

**Problem #3: The Flying-V Problem! (see appendix 5)**

After struggling through the long and challenging locker problem, I decided to implement a problem that we would only spend about 45 minutes working on and that was better scaffolded to meet the needs of all of my students. The problem I selected was the Flying-V problem which requires the use of patterns, especially as they relate to even and odd numbers. I included less
extension opportunities and instead reserved possible extensions on my desk on strips of paper for students who seemed ready to generalize the patterns they were finding algebraically. As in the other problems, I allowed the students to pick their own groupings and work in whatever space they felt comfortable (in my classroom or outside in the common area).

Students responded positively to this problem as they felt that it was more at their level (compared to the locker problem). Some actually complained that it was too “easy”; however, they encountered larger challenges with the extension opportunities (one of which was a scientific article on why birds fly in a v shaped pattern!) Upon introduction to this problem, the students got very excited because many of them were familiar with the v-patterns in which birds fly. In this problem I was able to walk around and try to take notes on what students were thinking. Many finished with the first part very quickly and so after about 5 minutes I was bombarded by students saying “is this right?” and “I’m done.” This was a bit frustrating because I was trying to engage in a conversation that was happening between a group of generally lower performing students who had drawn out beautiful pictures to go with the problem and noticed some fabulous patterns. One girl was very excited about the work that she had done and was voraciously trying to explain her pattern to her friend who was not following her excited explanation. Nevertheless, I had prepared some extensions on my desk and so I was able to get the students working on some algebraic endeavors for the problem while I continued on from group to group. Just as in other problems, I was very excited about the various approaches that I saw on this problem. I tried to make sure that I had students with various different strategies put examples on the board so that we could all see a variety of thought processes. Later I will discuss this further including pictures that I have taken of the board on this particular day. As in other days, there were still students who listed out all the numbers because they did not see a pattern.
After watching the other presentations, however, I think all students’ eyes were opened to various methods and shortcuts using patterns.

As far as next steps, I feel like the students need to keep working on paying attention to one another. This is part of the classroom culture that I worked to establish in the beginning, but am continuing to better throughout the year. The mathematical eloquence of many of my students is not very advanced and so I think that they often get frustrated as a student explains in front of the class. Nevertheless, I know that this is improving and is very important to the speaker and the listener. Maybe next time I will have the students prepare posters for their work on a given problem and then explain this to the class. My limited board space seems to be stifling these discussions! Secondly, I have noticed the importance of connecting each problem to something in students’ lives in order to spark interest. This is key to buy-in on the problem. I think this has been integral in each of my three problems thus far. Third, I like the freedom that students have in their work time space and groupings, however, I am thinking that for their next problem, I might have them work in their table groups to ensure that there is a diversity of thinking within each group. When the students pair themselves off, often there is a group of a few students who are able to take the problems to an extraordinarily advanced level of thinking and then a group of students who sits and works on the first part for the duration of the period. This is fine at times, but other problems could be very enriching in a mixed ability group. Finally, I would like to work on strategies to help these students get started with more ease; perhaps some sort of “hint” system would work. Maybe I need to go explicitly into some problem solving strategies (drawing pictures, looking for patterns, etc.) I mention these tactics regularly however have not gone over it explicitly and extensively. Perhaps this would be time well spent.
Overall the flying-v problem was successful because each of the students were engaged and were able to notice interesting patterns through a drawing and a sequence of numbers. Next time I would extend the problem to last for two periods and have the students make posters of their solutions. Because of the drawings, this problem definitely lends itself to this type of approach. The extended time period would also allow time for presentation of posters. I will keep the extensions as a task that is off to the side and available for students ready and not put as many options right into their journal. As I have seen, excessive word count in problems confuses the students and discourages them from working at it at all!

Mid-Semester Reflection

Thus far it seems that the students are enjoying open-ended problems on the whole. It is my next goal to determine if they like them because of the freedom of grouping, or if they are truly engaged in the problem. My suspicion is that it is some mixture of the two. In order to better determine this, I will not allow them to choose their own groupings on the next problem. I will switch between having them stay at their table with their group members or move elsewhere with their group members to see what effect freedom of space is having on their reactions.

As I go about analyzing my data, some trends that are emerging are that the students enjoy the problem the most if it is just at the very edge of their understanding (challenging, but not too challenging). The first problem seemed to be this way, but then the locker problem had a lot of complaints about it being too complicated. In response, I tried out a problem that was easier, but then I got some complaints that it was too easy. I hope that with the proper scaffolding that this problem will be at just the right level for all of the learners. I also have
noticed that their math confidence is improving as we go, and unlike the first two problems where they were slower to get started, students quickly got started on this most recent problem.

**Problem #4: The Checkerboard Problem (see appendix 6)**

Dealing with organization of information, pattern recognition, and perfect squares, the Checkerboard Problem evoked a variety of responses from the students. On this problem, I had the students work in their table groups which consist of a mixed gender and abilities. I also had all of the students stay inside the classroom and work at their tables. Prior to beginning the problem, I presented the students with some tips that they themselves had written for each other while working on these problems. (one of their journal prompt questions was “what tip would you have for other students working on this problem?) Here are a few of the tips that I extracted from their journals to show to each other. (see appendix 7 for the complete list of tips)

- “Some strategies I found were to observe my work carefully so I can see if there's any patterns and so I can try to find other different patterns.”

- “Some tips I would give is to draw out your thoughts and always have a reason for the answer you got.”

Receiving these tips definitely got the students focused on looking for patterns more readily than they had on previous problems though some got frustrated when they didn’t see the pattern right away. The complaining about the grouping requirements were minimal and the students got to work very quickly. It came to my attention early on that there were a few misunderstandings with the wording of the problem. Many oversimplified my wording and simply counted the obvious 1x1 boxes and in about one minute obtained the answer 64. I had to prompt their
thinking to explore other sizes of boxes which put them right back to work. Many students
wanted to find a pattern, but struggled with organizing their work and so they were unable to see
the patterns. I didn’t have sufficient remediations prepared to help with this situation. I also did
not allow ample time for presentation of the problem. I had an extension homework assignment
prepared; however, because of the rush I was unable to even hand this out. Furthermore, I had
the students take their journals home to write their reflections even though it had previously been
my policy that they would work on them only in class.

In this problem, I came to realize that it is very important to be familiar with your
problem before implementing it with your students for the first time. This enables you to be
prepared with extensions, remediations, and further questions for any and every need that arises
during the period. I know that I did not spend enough time thinking through this problem before
implementing it with my students and I think it showed. For this problem, a hint that involved a
“table” to help organize your information would have been a great help to some students. I also
realized that having the students work on their journal reflections at home actually had more
positive than negative come of it. The students seemed to spend more time and be more honest in
their reflection when doing it for homework. Working in their table groups seemed to go well
and I would like to continue to use this method on future problems.

**Problem #5: Gauss Gone Mad** *(see appendix 8)*

Earlier in the year, we learned briefly about the mathematician Carl Gauss. The students
read a story about how he amazed his teachers at a young age by adding the numbers from 1-100
in under a minute! The students investigated how he accomplished this feat using patterns and
had a great time! I brought back this concept for our work with integers for the Gauss Gone Mad
problem where the students were asked to add together the numbers from 1-100 with alternating
positive and negative signs. Since they had positive feelings about the first problem, they were excited to revisit the concept again and many even got out their old Gauss problem as a reference! In this problem, I did not require the students to work in their groups; however, I did ask that they stay in the classroom. There were other students working in the commons areas and I was concerned that this would be distracting to my students as they worked.

I initially thought they might not enjoy staying in the classroom, but it was actually the opposite. They welcomed the change and dove into the problem at their own desks! This grouping was very interesting because the students didn’t split up by ability and so some struggling students who might normally give up, were paired with students who worked very hard and explained their process throughout the entire problem and this helped boost their confidence. It was also interesting how some groups decided to split up the work. Some did this very effectively, however, being in a forced group actually may have led other groups down a less effective path. They felt like they needed to break up the problem into smaller pieces and start adding and when they did this, they then missed some of the patterns. Toward the end of our work time, I worked to find students with different strategies to have them put their work on the board and this seemed very beneficial. Unfortunately, many of the students who didn’t find the pattern often didn’t get the right answer (which I would hope they would to prove the point that all the ways work). Nevertheless, without finding the pattern, students were involved in lots of tedious adding and chances of doing this flawlessly are very slim.

I took a quick survey right after the problem asking students if they had a positive, medium, or negative experience on the problem and the results were overwhelmingly positive (see table 2).
I’m excited by this response as it seems like the kids are enjoying this type of problem even though the problem from today was not executed perfectly! I hope to continue to hone in on what is making this experience positive as we progress.

**Problem #6: Terrible Twos Problem (see appendix 9)**

In our work with order of operations, I came across some games where students are given numbers and they have to put them together using operations to come up with a set solution. I modified this idea to create the terrible twos problems. In this problem, students were once again given freedom of group and space to work. To begin this problem, I read it out loud and then gave the students a few minutes to work individually before letting them break up in groups. This was an attempt to help the students who normally fall behind in the beginning have some silence to think before beginning to work together.

Using this format, more students ended up staying to work individually than in previous problems. I think that they got a solid start on the problem individually and were too wrapped up in it to get up and move to work with their friends. Eventually, as they became stuck, they
became more social in their work. Despite these good strategies, the “terrible twos” problem is the first one that I have felt failed completely miserably as a result of my wording of the problem and not having thought through the process enough. I knew going into the problem that there may be some problems, but didn’t take the time to remedy my wording. The students were able to quickly able to come up with a way to make every single number using two, however they didn’t catch on that I wanted them to come up with the shortest method. What seemed like a good idea did not work because I did not develop it enough to keep the students entertained. I tweaked it a bit as we worked and they got more engaged, however I think I missed the crucial buy in part at the beginning of the problem. Our discussion the next day about finding the shortest way redeemed the problem, but I need to work on editing it to make it worded well from the start.

**Mid Semester Reflection**

I have now conducted five interviews and, though I don’t feel like the students have been speaking very eloquently about open-ended problems, I am definitely getting positive feedback. Every student that I have interviewed has expressed that they feel that these problems are both “real math” and are “more fun” than any other math that we do. My focus student Felice, who is one of the strongest math students I have ever taught, admitted that not every problem was challenging for her, but that she was nervous at times when she knew we were going to be working on a problem, and that there was a specific problem (locker problem) which was one of the hardest problems she had ever known! For a girl who has probably rarely been challenged in her mainstream math classes, I considered this to be a positive sign that these problems are in fact differentiating the curriculum to be accessible to both her and my struggling students. To
read more about my discoveries with differentiation and open-ended math problems, see
thematic findings below.

Another question that is emerging is with regards to the variety of problems that I have
been using. My first five problems dealt with looking for patterns to come to one solution and
my most recent problems have had an infinite number of solutions. I think that the students were
not prepared for this type of problem and therefore got more confused and didn’t see “the point”
of the problem. I will be taking both types of open-ended problems into account in my research
so it is important that I am noting any differences in their experiences on each kind.

**Problem #7: In Between Problem** *(see appendix 10)*

The in between problem was again open-ended in the sense that it had infinite solutions. I
like to begin my unit on fractions each year with this problem because it often starts the students
thinking about what is in between the integers. In asking the students to list all the fractions that
they can between 0 and 1, they are forced to think about fractions in a different way than usual
and you would be surprised how tricky this is for the students. I saw a variety of different
approaches:

- Some started at the very left of the number line and started with 1/100 and then
  moved as slightly to the right as they could, carefully labeling 2/100, 3/100, etc.
  Unfortunately what happened was they usually reached the other end of the
  number line then they were at about 47/100.

- Other students measured the line and then divided it into a set amount of pieces.
  They then made their fraction denominators equal to that. (example, the line was
8 inches long so they made each inch equal to 1/8 and progressed down the line that way)

- Other students started by putting ½ directly in the middle and then continued splitting from there into fourths, eighthths, sixteenthths, etc.

The latter method was the method that I preferred and that I gave hints toward if the students were stumped, however this and the previous method are both mathematically correct. Some students had an extremely hard time getting started and so this time I was prepared with some hints including “what is right in between 0 and 1” or, “how many pieces would you like to split this line into?” This seemed to work well and after about 15 minutes, I had some students with great strategies put their work on the board to spur students thinking even more.

One thing I would like to work on for this problem is the extensions. Some students had maxed out their line and had some extra time. I worked to give them extra ideas to think about, but I would like to make this process more formal for the next time I implement this problem, having strips of paper with extensions on my desk as in previous problems.

**Problem #8: Cookie Jar Problem** *(see appendix 11)*

The cookie jar problem deals with modeling with fractions and comparing the part to the whole in each case. In the implementation of this problem, I allowed the students once again freedom of space to work. This worked well for the cookie jar problem because many of the students worked at different paces though none of the students were able to finish so quickly that I didn’t get to walk around and check out their great variety of solution strategies! The extension for this problem was to make a poster for your solution of this problem and I was able to display
some truly beautiful work around the room! Students hung their posters up at the end of the period and the served as a resource on other occasions during our fractions unit.

On this problem, I was most astounded that, even after working on these problems routinely for three months, the students were still making positive comments. I read the cookie jar problem aloud to the class before passing it out and some comments I heard students’ utter were:

- “oooh, yes!”
- “I want to try it!”
- “Journal problems are fun!”

I thought my ears must be deceiving me because this enthusiasm about a math problem is not something I usually hear. Nevertheless, the students were anxious to get started, and I was impressed with their work as they attacked it like a puzzle.

From using this problem in the past, I know that work often begins with guessing and checking and many students think that they have the correct answer before they truly do because they mix up the part and the whole of the fraction. My advice repeatedly was to take their answer and run it back through the problem and then most students were able to see their blunder. Some students used the strategy of finding the least common multiple of all of the fractions and that seemed to work. I contemplated with the students (because I didn’t know the answer myself) if this would work if other fractions were substituted. It seems like an interesting connection that some students made and I think I will make it an extension for next year! I really liked the way that they were thinking outside of the box and tying together math concepts from throughout the year. Still other students approached the problem geometrically (some with a pie and some with a rectangle) and each of these groups of students were able to come to a correct answer as well.
This was a very positive experience both in groupings and in problem design. For next year, I would like to work on extensions as well as tips for struggling students for this problem.

**Problem #9: Coins Problem** *(see appendix 12)*

The coins problem was a closed problem that I manipulated into an open-ended problem. It had multiple answers and students were encouraged to come up with as many solutions as they could. It entailed work with money (beginning decimals) and expressing quantities as fractions (ratios). I was hoping that some students might attempt a more algebraic approach to solving the problem, however it turned out that the algebra was beyond their thinking and so most used a guess and check method (one student in each class came up with an equation that they thought might be useful but were unable to come to use it to obtain a solution).

Students had a very positive experience with this problem and seemed to enjoy employing the guess and check method, as if this was a puzzle that they were trying to complete. For homework, students were challenged to come up with another solution for the problem and at least ten reported back the next day with another solution! I was very proud of their diligence and that they were so positive.

As far as mathematical content goes, this problem was weaker than previous problems because the guess and check method made it more of a puzzle. Nevertheless, I think that the problem solving skills that it developed were invaluable to the students as they needed to organize their work and systematically guess and check to come up with solutions.

**Problem #10: The Magic Bag Problem** *(see appendix 13)*
The magic bag problem was the first open-ended problem that I completely made up on my own. I was looking for a way to reinforce the concepts of proper fractions, improper fractions, and simplifying fractions with the students and I had the idea that they could draw numbers from a bag and see if the outcome was one of three possibilities of numbers. The problem also delved into combinations/permutations and probability.

Perhaps it was the catchy, kid friendly outcomes, or the “magic” but the students had a great time on this problem and it seemed well differentiated to meet the needs of all of the students. On the whole, given one class period, most students made significant progress and finished the rest of the problem at home. The extensions were appropriate though no student made significant progress on them (which wasn’t bad since I myself hadn’t had much time to work on them before distributing the problem!) Some students had the idea of making actual pieces of paper and drawing out numbers (acting out the problem!) I thought this was a great idea, and when their final answers were slightly different from other students, it sparked an interesting conversation on the difference between the probability and actual outcomes. For next year, I would like to work with more of my students to get to the extensions because I think they solidified the content in the problem while offering the extra challenge of looking for mathematical patterns with numbers.

One bit of clarification that was needed on this problem was that students needed to change the fraction to simplest form before categorizing it. Another problem was that it was a bit verbose and so ELL students required extra support in understanding the meaning in the problem. I would like to work on making the problem more concise while keeping the content the same.
End of Semester Reflection

After completing ten cycles of problems, I have discovered a lot about how to design and implement them most effectively in my classroom. I have found the experience to be a positive one overall for the students and for myself. While methods of implementation should vary, problem design is key to the effectiveness of the problem for increasing problem solving and content knowledge. For a discussion on my overall findings throughout this process, please refer to my thematic findings below.

Findings and Actions: Part 2 (thematic)

In the following sections, results from my personal interviews are integrated with my analysis of student work samples, pre, mid, and post surveys, and interviews with six focus students. These findings are primarily written for teachers who intend to integrate open-ended math problems into their curriculum. Please see appendices 3-13 for the actual problems we used and for tips from my students to your students for working on these problems.

My findings about open-ended problems are structured into three themes that are loosely related to literature on my topic:

1. Students have a more positive perception about math and about themselves as mathematicians.
2. Students show increased problem solving skills
3. Students at all levels are challenged (differentiation)

Each theme is broken up into several sub-themes with evidence gathered from interviews, surveys, and work samples.
Theme 1: Students have a more positive perception about math and themselves as mathematicians

Sixth graders enter my classroom at High Tech Middle with many different perceptions of math. Some had elementary teachers who loved math and instilled a similar love in them. Others have yet to see the point. Each of these groups offer something special to my classroom and through open-ended math problems, it has been my goal to challenge each student, build confidence in each student, and show them that beyond all of the formulas and rules, math can be fun, social, and even beautiful. Through student interviews, work samples, and observations, I have carefully watched two perceptions change. First, the students are having more positive feelings about math in general, and secondly they are expressing more positive perceptions of themselves as mathematicians.

a. An enjoyment of math

Throughout the semester, students have expressed positive feelings about the math that they are working on. In a culture where math often gets a reputation as a rather dull subject, a student expressing sincere interest in a math problem is refreshing. As one eloquent student wrote, “I liked this problem because it was like a puzzle. The directions were like clues, hints to the answer. Every step you tried was like fitting a piece of the puzzle.” This enthusiasm has been echoed by students who expressed, “I was surprised when I saw the problem. It was a new one and I couldn’t wait to find it out.”

As the semester wore on and the students got used to our open-ended problems (we called them journal problems in class), I was anxious to see how their reactions would evolve. Sometimes something that is “cool” in the beginning of the year, loses its novelty after the fifth
or sixth go around. Nevertheless, after journal problems number seven and eight, I began hearing excited cheers from some students when they saw that a journal problem was on the agenda for the day.

Each class period that included an open-ended math problem began with me reading the problem aloud followed by me passing out the problem on a strip of paper to be glued into their journals. There was regularly a still silence as the students listened to me read the problem aloud. Some students would even write down information as I talked, eager to get an “early start”. In their journals, many students expressed their initial thoughts about these problems saying, “I was thinking that this (problem) would be fun! What I was also thinking was this problem looks like it’s going to take awhile and it might be hard.” Other students stated in their journals that, “I have never done a problem like this before so it was really interesting…” and “I really enjoy doing these journal problems and I think we should do them more often.” Table 3 below reflects students’ responses on an end of semester reflection. When asked if their overall experience on journal problems has been positive, medium, or negative, they responded as seen.

Table 3: Student responses on end of year survey
An overwhelming majority (78.7%) of the students responded in the positive about their journal problems. 14.9% of the students stated that some of the problems were positive while some were negative and only 6.4% indicated that their experience was mostly negative.

In my interviews, students have echoed the positive experiences discussed in their journals. A male student who is strong in math said that he “has really enjoyed these problems because they made him think a lot.” A girl who admittedly has struggles with math throughout her schooling said boldly, “They're different from just straight math, they're more funner actually.”

There were only a few students who indicated that their experiences were not positive. The explanation for their responses can be summed up in these two students’ quotes:

“It (my experience) was pretty negative just because most of them took a long time and I like to finish quickly.”

“I think that my experience on journal problems is negative because a lot of the problems are really hard… like the locker problem. Insane! That's not 6th grade math!”

Despite their negative responses on this final interview, each of the responders did have positive experiences on some of the problems and chose to specify a few problems that they did not favor for this response. Journal responses from earlier entries read, “This problem was pretty fun. It was very hard for me until I realized my misinterpretation,” and, “Yes I did like this problem because it was kind of challenging. That's what I like about stuff. If it's hard and I finish it, I'm proud of myself.”

Regarding the Locker Problem, the responses were more negative than on any other problem. Students stated that they had problems with the organization of the problem, with the
duration of the problem (3 days) and the trickiness of the patterns involved in the problem. After reading each of their responses each time, I worked to alleviate the specific problems that students brought up. No successive problem had the negative response and I think that some of the changes I made definitely made a difference.

Overall, students have had positive feelings about the open-ended math problems. I wanted to ensure that the positive experiences were happening for students across all ability groups so I decided to examine students’ experiences as they correlate to their pre-assessment score in my class. At the beginning and end of each school year, our school administers the MDTP diagnostic test which was developed by the UC/CSU system to determine the readiness of students’ math achievement from pre-algebra through calculus. My sixth grade students take the pre-algebra exam each year and scores are out of 40 points. As you can see in table 4 below, the overall positive experiences are consistent across both high and low pre-assessment scores indicating that these problems have been a positive experience for all ability groups.

Table 4: Student experiences weighed against MDTP score
I was worried that the positive experiences might come from students with higher abilities and the negative experiences might come from students with lower abilities, but this graph verifies that this is not the case. Kabiri states, “Providing open-ended problems helps teachers meet the needs of diverse learners since all students will benefit” and this is exactly what has been mirrored in my classroom throughout this semester (Kabiri, 2003).

b. Students see themselves as better mathematicians

As students have developed a more positive view of mathematics, they have similarly come to see themselves as better mathematicians. To investigate this, I asked students how they viewed themselves as mathematicians in a survey at the beginning, middle, and end of the study. (See appendix 1 for survey) See table 5 below for results.

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<th>Table 5: Student responses on beginning, mid, and end of year survey</th>
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The left graph is from the beginning of the year survey, the middle graph from the mid-semester survey, and the right graph from the end of year survey. I decided to pinpoint the students who marked “alright” on their initial survey with red to track their responses throughout the year.

It is clear that the students who marked “alright” made an increase from the first to the second survey in their feelings about themselves. It was during this part of the year that we were
working with open-ended math problems in our journal heavily (on a weekly basis). It is clear that in this transition, many students’ view of themselves as mathematicians did improve. From the mid to the final survey, there were unfortunately more students who went back to marking “alright” again. This actually correlates with a decrease in the amount of open-ended problems we were completing during this time period. As we took on our fractions unit at the time of the end of year survey, I went back to some more closed type of problem sets in an attempt to make sure the students got enough of the “basics.” I’m not sure how worthwhile this was, however it is evident that many students’ perceptions of themselves went back down a bit in this transition.

Overall, it is clear that the amount of students who marked “not very good at all” has decreased since the initial survey by at least three students, with only one student marking this in the end of the year.

In interviews with my focus students, I routinely asked them how they view themselves as mathematicians. Amber, my student who has always thought of herself as a “poor” math student reflected that now she is “in the middle” when it comes to her math skills. This year math has become easier because we do “projects” on it. When prompted what she meant by projects, Amber stated that she meant both larger projects and the journal problems. My other focus student, James, who has had similar struggles in mathematics reflected that these problem have made him “think hard” but that he was able to do them because he is an “OK” mathematician now. Though “OK” and “in the middle” seem like a weak attempt at confidence, to know these students is to know that they came into class feeling like they simply could not do math at all. A large part of their evolution has come through working on open-ended problems and feeling like they can succeed at difficult problems.

Theme 2: Students show increased problem solving skills
By working with open-ended problems, students in my class were able to increase their understanding while developing their problem solving skills. Evidence of this was scattered through their journals, expanded upon in class discussions, and spoken about in interviews. I saw their problem solving skills increase on four major fronts. The first was that they were taking complex problems and looking for patterns to make them simpler. Secondly, students began to see and embrace multiple strategies for solving each problem. Third, students started working collaboratively and embracing the help and opinions of students around them. And finally, students took on each problem with greater diligence and persistence, sometimes not giving up after an hour on one single problem.

a. pattern recognition

One major theme running through the students journals was patterns. The concept of patterns is very important in mathematics and sometimes math is even called “the science of pattern.” Throughout my teaching career, I have noticed that patterns are not often central to students’ learning of math at a young age. Because of this, I worked very hard to select problems that were tied to our current content, but that similarly focused on pattern recognition. Students picked up on this theme and regularly reflected on it throughout their journal entries.

“I learned that you need to keep looking for shortcuts and patterns and other ways that can get you through the problem quicker. I also learned that everything can maybe give you a clue (even your homework for example)”
“One tip is to write down (the number) each time and circle them when they are the same. Keep doing that and try to find a pattern.”

“I might think about patterns in a better way now which is a skill from this problem.”

Many students also came to realize that patterns will make a problem shorter. They admitted regularly to thinking that a problem was impossible, but then once they found the pattern, it was “easy.”

“I learned that in some math problems there are patterns. And when you find the pattern, you'll get the answer quicker. It makes it easy to solve the problem.”

“When you figure out a pattern that will work, STICK WITH IT! Before you doubt it and get another pattern, try out your first pattern.”

Open-ended problems are advocated by the NCTM because they require students to think about how they arrived at a certain answer as they focus on the process of thinking while weaving in mathematical content (NCTM, 2000). In working with these problems, students were able to step back from the core mathematical content and reason with patterns to arrive at conclusions. This stands in stark contrast to their experiences on problems void of any pattern recognition where students would sit back silently if they were unsure. Problems involving patterns gave the students a starting point and helped them to see math as more than just rules to be applied.
b. **Students develop multiple strategies to solve problems**

Another improvement in students’ problem solving abilities came from their ability to look at and appreciate multiple solution strategies for the same problem. Seeing multiple strategies for solving requires a lot of mathematical flexibility because students not only solve a problem, but communicate their reasoning to other students. Furthermore, they are listening to the reasoning of other students and either making sense of how their situation arrived at the same answer, or questioning why someone else’s strategy gave them a different answer.

One way that I had students display their different strategies was on the white board of my classroom. While working on these problems, I would routinely split the board up into three or four spaces. While I was rotating around and observing the students’ work, I would take special note of students who had unique strategies for solving the problems. As their reasoning developed, I would ask them to put their work on the board. When we reconvened at the end of the period, these students would stand in the front and share their approach while the other students listened on. After working on the problem for so long, the students were usually great audience members while their classmates presented (see picture of board below).
Instead of giving a midterm this semester, the students were assigned a journal problem and a partner. Together the two were to design a poster that outlined two very different methods of solving the same problem. These posters were presented jointly to the class and students were assessed on their ability to articulate the problem, explain their method and how it was different from their partner’s method. Some of the posters the students created are pictured below:

Examples of four of the posters made by students showing multiple solution methods for these open-ended problems.
As you can see from these pictures, the students successfully solved each of the journal problems and were able to highlight key differences from their two methods. According to NCTM, “Mathematics is something one does – solve problems, communicate, reason… it is not a spectator sport” (NCTM, 1989). As evidenced in their work using a variety of methods, the students are far from being spectators in my class. They are engaging with the content in multiple ways and articulately expressing their thoughts orally and in visually pleasing posters. As one student reflected, “Some math I learned was really just trying a bunch of things out… working together really paid off for me. I really learned to look for patterns someone else might not think of.”

c. Students work on math collaboratively

One of the key aspects of implementation of these problems was students working together. Often I would have a few minutes at the beginning of a problem for the students to gather their thoughts and sketch some initial ideas without the noise of others around them. After these few minutes were up, students had the option to work in the classroom or out of the class in the commons space on the floor with other students. As noted in the “description of intervention” section, there were a few problems where students worked in set groups; however, for most of these problems, they had freedom to choose who they would work with. The results with this were positive. Working collaboratively gave the students more confidence that they could solve these problems and they were able to tackle more challenging problems because they were putting their ideas together. As one student stated, “At first I was working alone… it seemed pretty impossible, so I started working in a group and it was much more productive.”
Another student echoed his feelings saying: “I learned that working in a group is easier because you get a lot more ideas and you could probably make a new friend. “

One student expressed their feelings about the group work by encouraging other students to take part in the process and not “be afraid” to share your work. This student, who is not often quick to share out answers was very proud of his work on this particular problem and the way that he was able to share some insights with the rest of the group. This was a turning point in the year for his confidence. He stated, “A tip I would give a friend is to definitely try every pattern you can think of and don't be afraid to tell your group because your idea might be the formula to solving the problem! Try strategies from other problems too!”

Some students expressed that it is important to choose wisely who you work with. She stated, “I learned that working in groups can sometimes be tricky when people are going faster than you.” Sometimes groups were moved around by me, however I was more impressed in my observations of students who were self-monitoring and moving themselves from bad work environments. Often students would get frustrated with the noise in the outside commons area and retreat to the classroom where there was often a quieter group of students working individually and then sharing ideas intermittently.

The pictures on the following page show my students working on their journal problems collaboratively. The first picture shows two students working together in my classroom (the quieter environment). As you can see, the girl is looking at the boys work to point out something that she noticed. The other two pictures show group work going on in our commons area. Students regularly expressed joy in getting to sprawl out and work freely. As long as they were being “diligent”, this was allowed.
Pictures show various groupings as students work on open-ended problems in my classroom and outside in our common space.
One student reflected the following on his group work experience, “Overall I really enjoyed this problem because it was challenging but once I figured it out, I felt great. It felt like I was in a science lab with some other friends as we analyzed the math and partied over our huge success of solving it.”

d. **Students develop persistence in solving math problems**

The growth in the students’ persistence can be summed up in this quote by a student: “I think the problem was easier because it only took us 25 minutes to figure out.” In my previous years of teaching, working on a problem for 25 minutes would be unheard of. Students routinely give up on problems after only a couple minutes of confusion. Amazingly, there were many quotes just like the one above sprinkled throughout my students’ journal responses on these problems. At first, the students were amazed that they only had one problem to work on for a whole period. After a few problems, however, they came to realize that this was normal for true mathematicians such as themselves. I explained to them that professional mathematicians sometimes work on one problem for many years in a row without ever coming to a solution. This impressed them, and over time, I began to see the students treating working on one problem for an extended period of time as “normal.”

One student offered the advice, “Don’t worry about doing this problem quickly or in one piece. Break it down into multiple ways and solve it step by step.” Many of the problems had multiple parts (discussed later in the differentiation section) and at first the students were intimidated by this. Soon they realized the progression and developed strategies for remaining diligent throughout the entire period.
A lot of students stated that they had initial confusion and what I appreciated about their follow up responses was that they didn’t allow themselves to stay confused. They stated,

- “At first I thought it was really hard and didn’t know how to do it. Then I finally got it and it was like a domino effect. Now I understand everything.”

- “It was really confusing at first but when I got it, it was really fun!”

- “I was confused when I read this, but I didn’t give up, I even tried it three times. I was surprised the answer wasn't 84 so I kept on trying.”

One of Steen’s arguments in his article, “How Mathematics Counts,” is that students working on closed math problems reason with numbers to produce an answer, however students working on open-ended problems reason about numbers to produce understanding (Steen, 2007). It is clear through my students work on these problems that they have developed a keen ability to reason about numbers for extended periods of times and without getting frustrated to the point of quitting. Through this, they have attained a seemingly deep understanding of the math that they were working on.

**Theme 3: Students at all levels are challenged (differentiation)**

One of the most striking phenomena to come out of this study was the way that open-ended problems were able to challenge students at all levels of understanding in my de-tracked classroom. Whereas some of my students are very savvy with variables, others still struggle with the multiplication tables. Because students at HTM come from all over the county and from over one hundred elementary schools, they enter my classroom at many different places mathematically. This causes tension at times in the curriculum and all math teachers have
various strategies for using this diversity of knowledge. Amazingly, the days where I didn’t feel tension in the curriculum at all was when we were all working on an open-ended math problem together. First, students at all levels admitted to “thinking hard” on a majority of these problems. Second, there was a very large variety of responses in what problems students found as most challenging and easiest overall.

a. **Students at all levels say that they are “thinking hard” about these problems**

My initial approach was to ask students if they felt “challenged” on these problems. Responses to this were interesting because they revealed that students didn’t think about being challenged in the same way that I was thinking about challenged. Many thought admitting to being challenged somehow revealed a weakness. Despite my observations that these problems were working and thinking very hard on these problems, students repeatedly stated in interviews and journal responses that they were not challenged because they had figured out the solution. It was almost as if once they figured out a solution method, they forgot about all of the work that they had done leading up to that solution. I brainstormed some terminology that might get at what I was really seeking from these problems, and together with my advisor, we decided that we would ask the students if working on these problems had made them “think hard.” This terminology was much more sixth grade friendly and a majority of the students stated that these problems did make them think hard. They elaborated on this by showing me that they were being “challenged” according to my definition of meeting each student at a place that is just beyond their current ability level.

Some students stated,
• “I liked the fact that it wasn't the easiest problem. This reminds me of Thomas Edison's saying ‘if we all did the things we are really capable of doing, we would literally astound ourselves.’ I feel it really helped me realize what I can do. “
• “Yes, the journal problems have made me work my brain to death. These problems make you think hard because they aren't like normal math problems where you don’t really have to think to answer.”
• “I was surprised that the math problem was just my level and that I was able to finish it before people put down the answer on the board. I was nervous about not being able to figure it out. I was happy about figuring out a pattern to come with the answer.”

My focus student Julio stated that these problems are “about the best level for everyone in the class except some people it might be too easy because there are a lot of really good mathematicians in the class.” He went on to say that “normal math seems to be just writing down a bunch of the answers and this is more of the thinking process, actually I spend a lot more time, like the locker problem I spent four pages of writing.” The student is reflecting on his thinking process as he worked on his math which echoes prior research stating that open-ended problems encourage students to think about how they arrived at their solution thus promoting a more mathematical disposition (Steen, 2007).
b. **Variance in responses to which problems were easiest and hardest.**

Throughout implementation of each of these ten selected problems, there were various students who excelled on any given day. This was not surprising considering the variety of problems that I used to implement. When surveyed regarding their opinion of the three easiest and hardest problems, it was amazing the distribution of answers that came up in the responses. As seen in table 6, there was a clear problem that had the most votes for “hardest” (the locker problem) and a clear problem that had the most votes for “easiest” (the coins problems). Nevertheless, each of the aforementioned problems had some votes for the opposite direction: the locker problem had two votes for the easiest and the coins problem had five votes for the hardest.

*Table 6: Student opinions on difficulty of problems*
The ramifications of table 6 were that on any given day of implementation, all students were being challenged at different levels, making some students experts one day and other students experts the next. This helped to alleviate the idea that is perpetuated in some classrooms that a couple of students are always the “best” and those who struggle will always struggle.

Conclusions

In my study, students engaged in open-ended math problems. Open-ended math problems include both problems that can be solved in various ways and problems that have many different solutions. They worked on these problems in various group settings including some teacher-determined groups and other groups freely chosen by the students. They also worked on these problems with a varying amount of support. On some occasions, I offered hints from the beginning and at other times, I asked the students to simply support each other in their work. It was my goal to document and analyze students’ experiences with these problems and the success with which these problems engaged and challenged each of the students in my class. At the end of my semester of study, I carefully analyzed my data to look for trends in the students’ responses both quantitatively and qualitatively.

From surveys and quantitative data, students’ experiences with open-ended math problems were overwhelmingly positive. It was further revealed through my students’ comments that they were expressing positive perceptions about math and about themselves as mathematicians as they worked on these problems. Though there were some nervous feelings initially, students articulated sincere enjoyment for the freedom they were given and the challenges that these problems gave them. In our problems later in the semester, students expressed excitement to get to work on a “journal problem” on a particular day. Similarly,
students overall showed an improvement in how they described themselves as mathematicians. Some struggling students gaining the courage to say that they were “OK” at math and less students saying that math is their worst subject. According to the NCTM in their Principles and Standards from 2000, “Mathematical competence opens the doors for productive futures.” They go on to state that everyone needs to understand mathematics so that they can have an opportunity to “shape their futures” (NCTM, 2000). As open-ended problems improved my students’ perceptions of themselves as mathematicians, they opened the door to the world of mathematics for them and, for many, piqued their interest in this very important subject area. By gaining a more positive view of the subject area and of themselves as experts in the subject area, students are more likely to continue their study in this field throughout high school and college.

Further interviews and work samples showed that my students had increased problem solving skills due to open-ended problems. One measure I used to determine this was pattern recognition. In his later work, well-respected mathematician, George Pólya stated that pattern recognition is one of the most significant strategies used by successful mathematicians (Pólya, 1957). Through work samples and interviews I was able to see many signs of such pattern recognition and therefore increased problem solving skills in my students. Secondly, my students’ problem solving improved as they learned to look for and appreciate multiple strategies for solving the same problem. According to the NCTM, “Good problem solvers have a ‘mathematical disposition’” (NCTM, 2000). In searching for multiple strategies for various problems, my students moved toward this analytical “mathematical disposition” which drastically increased their problem solving skills. Finally, my students’ problem solving increased as they attained an openness to collaboration with one another. This correlates with Schoenfield’s research where he found that mathematics is largely a social enterprise. He stated
that, “Many problems considered central are too big for people to solve in isolation. In consequence, an increasingly large percentage of mathematical and scientific work is collaborative. Such collaborative work both requires and fosters shared perspectives among collaborators in particular and across the field at large” (Schoenfield, 1994). The magnitude of these open-ended problems engaged the students and they began to rely on each other, just as the mathematical and scientific community does as they work together to solve large problems.

Finally, my research on open-ended math problems produced quantitative and qualitative data revealing that students at all levels of mathematical ability were challenged. This shows that these problems were an equalizer and an extremely valuable tool for differentiation in my classroom. This finding correlates with the work of both Hertzog and White who stated that curricula for the gifted and talented should be in depth and complex while curricula for struggling students should be based on real life situations with a decreased emphasis on the correct answer (Hertzog, 1998; White 1997). Open-ended problems catered to both of these needs and, as shown in my classroom, met the needs of all levels of students.

**Implications**

The broader implications for these findings point toward better math education in all classrooms both in my school and at any school willing to implement them. The National Council of Teachers of Mathematics has worked hard in the past two decades to reform math education to better meet the needs of our society through increasing problem solving skills and promoting the application of math concepts. Open-ended math problems are clearly a useful tool as, by design, they are an application of concepts, and, as shown in my research, they increase
student problem solving abilities in many ways. Also, as math is a subject that many students “turn off” to at a young age, open-ended problems have been shown to develop intrigue, excitement, and engage all students in a problem solving endeavor. The problem solving skills that are developed through open-ended problems would be immensely useful in any math classroom, used as a supplement to current curriculum, or as a basis for a new curriculum. They are particularly useful for schools like mine that do not track based on ability because of the problem’s ability to differentiate the curriculum for all students. If math classrooms throughout the district, state, and country could begin to implement these types of problems, even on a semi-regular basis (2-3 times per month), I think that there would be positive implications for math programs and on the readiness of math students for larger society.

In order to maximize the usefulness of these problems, here are some methods that I have found most useful in my classroom.

**Tips for Teachers:**

**Solve the problem first yourself.** It is important, before you give a problem to students, to be aware of the basics of the problem so that you can help struggling students gain a solid foundation. Similarly, it is good to know some related extension exercises to deepen students’ understanding.

**Encourage group work and conversation.** Because open-ended problems allow multiple approaches, they offer an ideal context for students to learn from each other and share responsibility for finding solutions.
Allow freedom of movement. When we work on these problems, I allow my students to go anywhere in my room or our common spaces to work. Sometimes a change of scenery is what they need to jump-start their thinking.

Refrain from giving hints for a set period of time. Some students are apt to give up the moment they feel confused. By holding off on giving hints, you are encouraging them to “think for themselves,” and this is where immense growth happens. After a set time, offer support by encouraging them to draw and label a picture of the scenario. After this, you could pair them with a partner or group who seems to have a good start on the problem and who can offer some advice.

Hold off on telling them if their answer is “right” or “wrong.” If students say that they are done, I like to ask them how much they would be willing to wager on their answer. This gives me an idea of how confident they are in their thinking and encourages “guessers” to keep working at it. Another question I have posed with students is “could you solve this another way?” which often sets them off to work again trying to find other methods to “check” their solution.

Have students post and explain work on the board. Articulating their process is often the hardest part for early finishers. Encourage them to work out their solutions on the board in such a way that younger students would understand their thinking. Sometimes even incorrect approaches are helpful to put on the board so students can see where their thinking went wrong.
As I have examined my students, I have deliberately neglected gathering quantitative data of their progress on understanding content on more traditional exams as it relates to their work with open-ended problems because I wanted to focus on their experiences. Though I know that an increase in problem solving skills and confidence will help them on any exam or standardized test, an interesting question would be to see exactly how the knowledge that they gain on open-ended problems translates to more traditional math exams. Another area for continued research would be in the development of these problems. Problems could be designed that start with a sixth grade base of understanding and extend to twelfth grade content because of the connected nature of mathematics. It would be great to see a group of teachers work collaboratively to design problems that were accessible, challenging, and covered content from so many grade levels.

In a time where many students are being turned off to the field of mathematics, open-ended problems offer a fresh new look into the world of math. By piquing curiosity and revealing the beauty and usefulness of the field, open-ended problems have revealed that they can turn students on to math and help them to develop key problem solving skills that will be an asset in a plethora of jobs. For teachers, open-ended problems offer a tool for natural differentiation and connectivity between math concepts in the classroom. Used well, open-ended problems are an exciting possibility for improving mathematics education.

**Final Reflection**

I began my research process extremely interested in open-ended problems and am concluding my research similarly intrigued by their role in the math classroom. My view of their
place in the classroom has been refined through this experience and my overall perception enhanced as I have attempted to look beyond my personal opinion of them and see their effectiveness through my students’ eyes.

The research process has primarily impacted me in the way that I view my students’ feedback in my classroom. I have always worked hard to balance my summative and formative assessments and use the results from such assessments to gain a better understanding of my students’ level of knowledge on a given topic. Through this process, however, I have learned to daily gauge not only my students’ understandings for the math that we are doing, but how they are feeling about the tasks at hand. Throughout my research I came to see the large role of students’ self-efficacy and the role that this plays in their ultimate success and so I have worked to make this part of my assessment procedures. In my classroom I regularly ask myself questions like:

- What are my students bringing to the classroom today that might impact their learning and how can I help take care of these needs?
- Are any students sitting around with nothing to do because they finished early?
- Are any students so confused that they don’t know where to start?
- Do the students seem excited about what they are working on?
- Are the students talking to one another about their work?
- Are students talking to me about their work?

By gauging my students’ feelings and experiences with the above questions on a daily basis, I have learned to tailor my activities, sometimes even changing plans in the middle of a lesson to make sure that I am meeting all of the students’ needs. This has been a positive
outcome of being a researcher in my own classroom, and I know that it will positively impact my teaching in the future.

In addition to gauging students’ feelings, I have found better tools to assess my students’ level of understanding. Modeled upon my research methods, the use of quick surveys and checks at the end of a period have streamlined these assessment efforts. These can be as simple as a blind survey where the students put their heads down and hold up a number to rate the difficulty of assignment on a scale of 1-5. They can also be as formal as an exit slip with a few problems designed to identify any misconceptions from the day’s activities.

Both in their feelings and their understandings, the action research process has taught me the value of student feedback and helped me to develop methods of quickly and daily achieving useful feedback. This is an invaluable tool that will undoubtedly be key to effective teaching throughout the rest of my career.

Whereas many researchers find their question and focus evolving throughout their research process, my question, “How do students’ experience open-ended math problems” did not change at all from the beginning of my study. I was pleasantly surprised by this and attribute it to the simplicity with which my study started. As stated in previous sections, I entered this research project having already been intrigued by open-ended problems and having already tried a fair amount of them in my classroom. Because I had heard teachers and mathematicians tout the benefits of these problems, I therefore made the focus of my study not their “perceived effectiveness though my eyes” but rather their effectiveness through students’ eyes. In focusing solely on students’ experiences, I was able to hone my methods around the literature that I read and I never had to change my question. Regardless of how my students’ experienced the problems, I continued to gauge their experiences through daily checks,
interviews, and reflections in accordance with my pre-determined methods. The evolution that happened, rather, was in my processes of implementation and that is how I was able to come up with several tips for teachers for how to effectively implement these problems.

One area of change that happened before my study got off the ground was my method of attaining verbal feedback from each of the students. My initial plan, as part of my methods of data collection, was to incorporate focus groups. After beginning my research, I opted for individual interviews in lieu of the focus groups because I thought that I would get more authentic responses. Looking back, however, I feel that focus groups could have been a valuable experience. In individual interviews, I felt that the students sometimes felt inclined to say what they thought I wanted to hear, despite my prompting that these interviews would not affect their grade or my opinion of them. In a group, there would have been the chance that students would feed off of each other and foster a group-think atmosphere that would not truly represent each of their individual opinions; however I now think that I would have liked to hear some of their thinking in the group setting because I may have heard more detailed responses as they listened and responded to each other.

Something else I would do differently is gather quotes from my students as they worked. Students regularly said very interesting things while working on these problems, but I found it hard to balance assisting them as needed and recording the things that they said. While I mentally recorded their talk and used this information to inform my thinking, it was not explicit and so I was unable to incorporate this into my writing. One way to help this situation for next time would be to stage recording devices around the classroom to record my students’ speech while they worked. This could further shed light on group dynamics during their work time which would be another interesting avenue to investigate.
For their select open-ended math problems, my students worked in a math journal. This was my plan all along and I was pleasantly surprised with how it turned out. I now think that I will have my students keep a math journal every year. One of the notable benefits was that students saw these problems as “different and important” not just another hard problem that they might see on a daily basis. Having them take out a math journal seemed to encourage the students to take the problem that was to come more seriously. In my research process, these were an effective way of keeping the students’ work samples in one place. These journals most often stayed in the classroom and anytime I wanted to refer to a specific student’s response to a given problem, I could sift through the pile and find their journal and investigate.

After each of their problems, I had the students complete a written reflection on the problem on the opposing page. This was a valuable practice for my research, however I don’t think that I would continue this practice for every problem in my future teaching. I do, however, see the value of students’ writing in math and this is definitely something that I will continue to incorporate for certain problems and assignments. Writing across the curriculum is something I see as extremely important and this was one way that I was able to do that this year and I think the students reaped the benefits.

Most challenging in the research process was time management. As most teachers do, I always want to produce engaging math assignments and projects. As a researcher, I wanted to be sure that I wasn’t missing out on any lost opportunities for gathering data, quotes, interviewing, and gathering work samples. Often these two hats were able to be worn at the same time; however, sometimes they conflicted because of time. For these past two years of working formally as a teacher and a researcher, I have struggled to make ends meet time-wise and find the proper balance between the two goals above. As I move out of my official role as researcher, I
hope that I never lose the drive to see my classroom as a mini-lab where I am constantly engaging with my students to design curriculum that will meet their needs and make them feel important. Nevertheless, I realize that I will have to streamline the “researching” facet so that it flows almost seamlessly out of my teaching. After two years of formal researching, I have some ideas for bringing the teacher and researcher together to make them feasible time wise:

- Always listen to your students and take what they say seriously
- Use mini understanding checks like exit slips and quick chats to assess students’ understanding daily
- Always be encouraging, particularly to students who you know to struggle with self-confidence in the classroom
- If you see that a lesson isn’t going well in the middle, change it right then to meet their needs

In the future, I will definitely continue to use open-ended problems, giving them a central place in my curriculum. In addition to this, I would like to continue to develop more of these problems to share with my colleagues in all of the middle school grades. Because many of these problems have the ability to extend into algebra and beyond, it would be awesome to have some problems that are known across the grades and are extended upon each year. This would truly show the connectivity between math throughout a students’ school experience and could be a great professional development opportunity for math teachers in a school to work on. I would furthermore like to develop a resource for teachers beyond the scope of my own school. This resource would not only have problems and associated tips and challenges, but also various solution methods from students and possible responses to each of these solution methods.
Because I found it so imperative to be familiar with the problem before implementation, this would take some of the footwork out of this task for busy teachers.

The conclusion of my Action Research project has brought to light many of the inner workings of my classroom. In my attempt to see math through my students’ eyes I have gained insights for effective instruction that will no doubt benefit me and any math teacher I work with in the future. I realize that it is only by understanding how students experience math in my classroom that I can design open-ended math problems that are useful, enjoyable, and effective in helping students improve their problem solving skills, make connections in math, and therein see a glimpse of the beauty that this subject can hold for everyone. I hope that through my continued research I can help math teachers work together to ensure that fewer students walk out of our classrooms saying, “Oh, I’m just not a math person,” and that more feel empowered to take on any problem that comes their way.
References


Appendix 1: Pre/Mid/End Survey

1. How would you describe yourself as a math student?
   a) I’m a math genius…give me any problem and I’ll solve it for you.
   b) I’m pretty good at math… it’s my best subject.
   c) Math is alright…I am improving at it.
   d) I don’t think I’m good at math at all
   e) Other ____________________

2. How did you feel when you initially read our problem for today?
   a) Excited to try it out such an interesting problem
   b) Bored, just another math problem
   c) Nervous, but hopeful that I would be able to figure it out
   d) Terrified that I wouldn’t know the answer
   e) Other ____________________

3. What part of the problem was most challenging for you?
   a) the initial model/set-up
   b) figuring out how to use the information I was given
   c) coming up with my final answer
   d) presenting my answer to the class
   e) understanding why I have to do this problem

4. What do you feel your grade should be for this open-ended problem?
   Great job with the scale here!
   F …………..D ……………C……………..B……………..A

   I didn’t attempt any model or any work. I don’t know what the problem was about.
   I did OK. If I didn’t understand something I asked the teacher right away. My answer is probably not right.
   Awesome, I worked hard, modeled, presented, and am confident in my answer

5. Rank the following math groupings from 1-4 (1 being you prefer it over all others, 4 being it is your least favorite)
   ___ Group work/Group quizzes
   ___ Partner brainstorm/activities
   ___ Individual work time/ Individual quizzes
   ___ Presenting work in front of the class
Appendix 2: Math Journal Requirements

Math Journal Requirements

- All *math work* is to be done on left pages. Right pages are reserved for *written responses*.
- All pages should be numbered (table of contents should be kept up to date)
- Show all work for each problem (label where necessary!)
- Label each part of the problems separately (example: part a, part b, part c)
- Draw a box around any final answers.
- Working together is awesome, however each student’s journal needs to reflect their work on that particular problem.

Math Journal: Written Responses

1. How did you feel when you initially read this problem?
   - I was surprised that…
   - I was nervous about…
   - I was happy about…

2. What did you learn from this problem?
   - Some math I learned was…
   - Some strategies I learned were…
   - Next time I would like to…

3. What are some tips you would give a friend about this problem?
   - First you should…
   - Don’t worry about…
   - In order to solve this problem quickly you could…
   - Here are four quick steps for solving this problem.

4. Overall did you like this problem?
   - This problem was hard/easy/medium because…
   - I would/would not recommend Mrs. Strong use this problem again because…
   - I will/will not use the information/strategies I used in this problem again because…

If you have more time, respond to the following questions:
- What do you like about math? What don’t you like about math?
- Is math your favorite subject? Why or why not?
- What is one math topic that you wish you knew more about?
- Do you feel that you have grown as a mathematician this year?
Appendix 3: Journal Problem #1

The weather report problem
The weather is reported every 9 minutes on ABC and every 12 minutes on CBS. Both stations broadcast the weather at 1:30. When is the next time the stations will broadcast the weather at the same time?

a) When was the last time (before 1:30) that the weather was reported at the same time?

b) List the next 5 times that the weather will be reported at the same time. How often does this happen? Explain.

c) Suppose CBS changed to reporting the news only every 24 minutes. When is the next time that they will report the news together?

d) Since CBS has been getting low ratings on their weather program, they have decided to only report the news every 45 minutes. After 1:30, when will they report the weather together again?

e) Are there any numbers you could choose that would make it so that the news programs were never reporting the news at the same time?
Appendix 4: Journal Problem #2

The Locker Problem
Imagine High Tech Middle decided to install lockers for each of the 300 middle school students. The lockers are numbered from 1 to 300. When the High Tech Middle students return from summer vacation, they decide to celebrate the lockers by working off some energy.

- The first student goes to all of the lockers and opens every locker.
- The second student then goes along and shuts every other locker.
- The third student changes the state of every third locker. (if it’s closed, they open it, if it is open, they close it)
- The fourth student changes the state of every fourth locker.
- The fifth student changes the state of every fifth locker, the sixth every sixth locker, etc…

a) Let’s start by looking at the first ten lockers. After ten students go marching, which of these lockers are still open?
b) Now let’s look at ALL of the lockers. Imagine that this pattern continues until all 300 students have followed the pattern with the 300 lockers. Which lockers will be open and which will be closed when they finish? Why?
c) What if the students were not in order when they went marching to open and close lockers. Would the same lockers be open in the end? Why or why not?
d) Which lockers were only touched by two students? Were any lockers touched by only three students? Which number locker was touched by the most students?
Appendix 5: Journal Problem #3

The Flying V! Problem
Groups of ducks often fly in a v pattern as seen below. The first three possible flying v patterns that birds could fly in are seen here:

a) Draw the next 5 v-patterns. How many birds are in each?
b) How many birds will be in the 100th v-pattern? Justify your answer using pictures, tables, explanations, etc.
c) Which v-pattern will have 1000 birds in it? Explain.
Appendix 6: Journal Problem #4

The Checkerboard Problem
Macy and Nick were playing checkers when Macy suggested, "Let's count all the squares on the checkerboard!"
Nick said, "That's easy, all we have to do is multiply 8 X 8."
"Oh no, Nick. There are many more… and lots of different sizes," Macy responded.
Help Macy and Nick count all the squares on the checkerboard.
Appendix 7: Tips from the students

1. Don’t get frustrated… it’s ok to solve the problem in different ways from your friends.

2. Don’t just read it and think “Oh, this is too hard” and then goof around. Once you actually think about the problem, then you get it.

3. Don’t stress out and remember what math is: patterns!

4. A good way to start is to draw out your thoughts.

5. Always have a reason for the answer you got.

6. These problems can seem endless but once you get them, you always remember them.

7. Don’t worry about doing this problem quickly or in one piece. Break it down into multiple ways and solve it step by step.

8. Don’t worry about anyone who is finished before you.

9. Try every pattern you can think of and don’t be afraid to tell your group because your idea might be the formula to solving the problem!

10. Try strategies that you used in other problems.
Appendix 8: Journal Problem #5

Gauss gone mad!
We all remember our friend CARL FRIEDRICH GAUSS (he was the famous mathematician who quickly added the numbers from 1 to 100, amazing all of his teachers!!!). Today, Gauss has an extra challenge for you. He wants you to add the numbers from 1 to 100 again, but this time with alternating signs. Here is the beginning of your list of numbers:

1, -2, 3, -4, 5, -6, 7, -8, 9, -10, 11, -12, 13, -14, 15, -16, … , 99, -100

What is the sum of all of these numbers? Use your knowledge as an integer wizard to look for shortcuts, patterns, and the sum!

Extension 1: Does it matter that the odds are negative and the evens are positive? Check the sum of these numbers.

-1, 2, -3, 4, -5, 6, -7, 8, -9, 10, -11, 12, -13, 14, -15, 16, … , -99, 100

Extension 2: Suppose Gauss asked you to find the sum of all of the even numbers with alternating signs? (2, -4, 6, -8, 10, -12, 14, -16, 18, … , -98, 100)
Would the sum again be -50 just like in the first problem? Why or why not?

Extension 3: Check the sum of sequences with multiples of three, four, and five. Do you notice any patterns?
Appendix 9: Journal Problem #6

The “Terrible Twos” problem

“My favorite number is TWO!” shouted Tyler.

“Why?” asked Trevor, “There’s nothing special about it.”

“That’s not true. You can make any number in the whole world, using just twos… and a few operations, of course,” Tyler argued back.

“I don’t believe you… prove it!” responded Trevor.

With his head held high, Tyler began, “Ok… well 1=2÷2 AND 2=2+ (2-2) AND 3=2+(2÷2) AND…”

Do you think that Tyler is correct? Can you generate EVERY integer from just TWOS and a couple of operations? Try to find the simplest/shortest way to make the numbers 1-100 using only TWOS.
Appendix 10: Journal Problem #7

**The “In Between” Problem:** On the number line below, label as many possible numbers that you can between the integers 0 and 1 (*please do not use decimals for this activity, but DO put them in the correct location*):
Appendix 11: Journal Problem #8

Cookie Jar Problem: There was a jar of cookies on the table. Daniel was hungry because he hadn't had breakfast, so he ate half the cookies. Then Tannia came along and noticed the cookies. She thought they looked good, so she ate a third of what was left in the jar. Lisette came by and decided to take a fourth of the remaining cookies with her to her next class. Then Hannah came dashing up and took a cookie to munch on. When Avery looked at the cookie jar, he saw that there were two cookies left. "How many cookies were there in the jar to begin with?" he asked.
Appendix 12: Journal Problem #9

The Coins Problem
I have twenty four coins in my pocket. I count up the total and it adds up to $3.78… enough to buy an awesome lunch! What fraction of the coins in my pocket are quarters? What fraction of the coins are dimes? Nickels? Pennies? How many answers are there to this problem?
Appendix 13: Journal Problem #10

The magic bag problem
A fortune teller has a magic bag with the numbers (1, 2, 3, 4). Two numbers are chosen at random from the bag by a man in the audience. The two chosen numbers are used as numerator and denominator of a fraction. The fortune teller says that if that fraction that is made is equivalent to a whole number, the person will have a good life forever. If what is drawn is an improper fraction, the person will have something bad happen to them very soon. And if the fraction is proper, then the person will grow wings and fly away.

- What fraction of the possibilities will produce a good life forever?
- What fraction of the possibilities will cause something bad to happen?
- What fraction of the possibilities are cause wings to grow and fly away?
- Would this man’s odds stay the same if he were given the numbers (1, 2, 3, 4, 5)? What about (1, 2, 3, 4, 5, 6)? Is there a pattern here?